

# How are glories formed?

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Mie theory can be used to generate full-color simulations of atmospheric glories, but it offers no explanation for the formation of glories. Simulations using the Debye series indicate that glories are caused by rays that have suffered one internal reflection within spherical droplets of water. In 1947, van de Hulst suggested that backscattering (i.e., scattering angle  $\theta = 180^\circ$ ) could be caused by surface waves, which would generate a toroidal wavefront due to spherical symmetry. Furthermore, he postulated that the glory is the interference pattern corresponding to this toroidal wavefront. Although van de Hulst's explanation for the glory has been widely accepted, the author offers a slightly different explanation. Noting that surface waves shed radiation continuously around the droplet (not just at  $\theta = 180^\circ$ ), scattering in a specific direction  $\theta = 180^\circ - \delta$  can be considered as the vector sum of two surface waves: one deflecting the incident light by  $180^\circ - \delta$  and the other by  $180^\circ + \delta$ . The author suggests that the glory is the result of two-ray interference between these two surface waves. Simple calculations indicate that this model produces more accurate results than van de Hulst's model. © 2005 Optical Society of America  
*OCIS codes:* 010.1290, 290.4020, 240.6690.

## 1. Introduction

Glories are caused by backscattering of sunlight from small droplets of water. As surface tension ensures that small droplets are spherical, Mie theory<sup>1</sup> can be used to produce full-color simulations of atmospheric glories.<sup>2,3</sup> A companion paper<sup>4</sup> in this issue uses Mie theory calculations to examine the appearance of glories as a function of droplet radius  $r$  taking into account factors such as the size distribution of the droplets and the effects of polarization.

Despite the successful application of Mie theory to the glory, there is no simple explanation for the formation of glories. This paper reports the use of the Debye series to identify the ray paths causing the glory and, subsequently, quantifies the contributions made by surface waves to the scattering processes.

All of the graphs and simulation in this paper have been generated using the MiePlot computer program developed by the author. This program can be downloaded free of charge from <http://www.philiplaven.com/mieplot.htm>.

## 2. Debye Series

The Debye series<sup>5–7</sup> can separate the contributions made by light rays of order  $p$ , where  $p = 0$  corresponds to external reflection and diffraction,  $p = 1$  corresponds to direct transmission through the sphere,  $p = 2$  corresponds to one internal reflection,  $p = 3$  corresponds to two internal reflections, and so on. For  $p \geq 1$ , the number of internal reflections is given by  $(p - 1)$ . Although it is well known from geometric optics that  $p = 2$  rays form the primary rainbow and that  $p = 3$  rays form the secondary rainbow, the Debye series goes far beyond the limitations of geometric optics by including the effects of diffraction and surface waves. It should be noted that the Debye series is not an approximation: the summation of the Debye series for all integer values of  $p$  from zero to infinity gives the same result as Mie theory.

Figure 1 shows curves of intensity calculated using Mie theory and the Debye series for scattering of sunlight from a spherical drop of water with radius  $r = 10 \mu\text{m}$ . In this paper, the term “sunlight” implies calculations based on a light source with an apparent angular diameter of  $0.5^\circ$  with the spectrum of sunlight between 380 nm and 700 nm as recorded at ground level by Lee.<sup>8</sup> The Debye  $p = 2$  term (corresponding to light that has suffered one internal reflection in the sphere) is dominant in forming the colored rings of the glory. For scattering angles  $\theta < 179^\circ$ , the intensity of the  $p = 2$  term is at least one order of magnitude greater than that of other Debye

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Received 3 January 2005; revised manuscript received 17 April 2005; accepted 18 April 2005.

0003-6935/05/275675-09\$15.00/0

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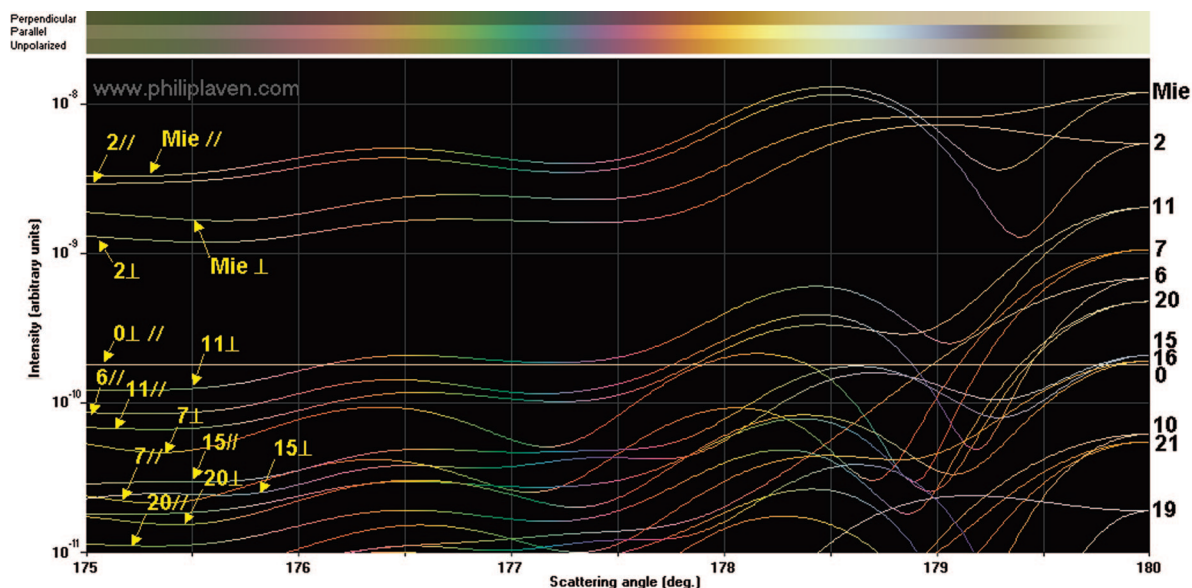


Fig. 1. Comparison of Mie-theory and Debye-series calculations for the scattering of sunlight by a spherical water drop with radius  $r = 10 \mu\text{m}$  ( $m//$  and  $m\perp$  denote the Debye-series terms  $p = m$  for polarization-parallel and perpendicular to the scattering plane, respectively). The colored bars above the graph represent the brightness and color of the scattered light calculated using Mie theory, while the curves in the graph represent the saturated color of the scattered light.

terms; thus the curves for the  $p = 2$  term and for Mie theory are very similar in this region. However, as  $\theta \rightarrow 180^\circ$ ,  $p = 11$ ,  $p = 7$ ,  $p = 6$ , and higher-order terms make substantial contributions to the white central feature of the glory.

For  $\theta < 179^\circ$ , the polarization parallel to the scattering plane is dominant for the  $p = 2$  and  $p = 6$  terms, whereas the polarization perpendicular to the scattering plane is dominant for the  $p = 11$  and  $p = 7$  terms. Note that, with the exception of the  $p = 0$  curve (which is effectively white), the other curves in Fig 1 show almost identical colors as a function of  $\theta$ , such as the reddish colors around  $\theta = 177.6^\circ$  and  $176.2^\circ$ . This is unlikely to be a coincidence!

Figure 2 shows two simulations of the glory caused by scattering of sunlight from a spherical droplet of water with  $r = 10 \mu\text{m}$ : the left side shows a simulation based only on the Debye  $p = 2$  contribution, while the right side is based on Mie theory. The key difference is that, as indicated in Fig. 1, Mie theory predicts a bright white zone at the center (i.e., for  $\theta \rightarrow 180^\circ$ ). More importantly, both simulations produce essentially identical sequences of colored rings. As the intensity of the  $p = 2$  contributions shown in Fig. 1 is, at least, an order of magnitude greater than the intensity of other contributions, Fig. 2 indicates that the colored rings of the glory are primarily caused by light that has suffered only one reflection within the water droplet.

### 3. Explanations for the Glory

As the above Debye-series results indicate the importance of  $p = 2$  contributions, it is appropriate to consider the geometric ray paths that might cause the glory. The basic geometry is illustrated in Fig. 3. The

scattering angle  $\theta$  resulting from a ray with angle of incidence  $\alpha$  on a spherical droplet is defined for an arbitrary value of  $p$  by

$$\theta = (p - 1)180^\circ + 2\alpha - 2p\beta, \quad (1)$$

where  $n$  is the refractive index and  $\sin \beta = \sin \alpha / n$ . For  $p = 2$ , this simplifies to

$$\theta = 180^\circ + 2\alpha - 4\beta. \quad (2)$$

Figure 4 shows the results of calculations using geometric optics for  $p = 2$  rays and for radius

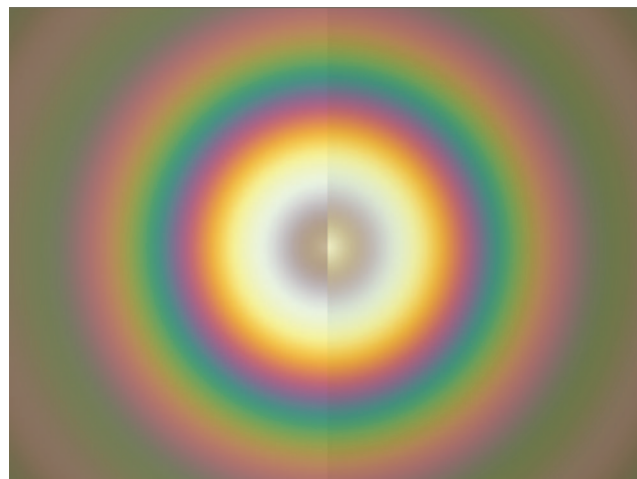


Fig. 2. Comparison of simulations of the glory caused by scattering of sunlight by water drops of radius  $r = 10 \mu\text{m}$  using Debye  $p = 2$  term (left) and Mie theory (right) The width of this image corresponds to an angle of about  $\pm 5^\circ$ .

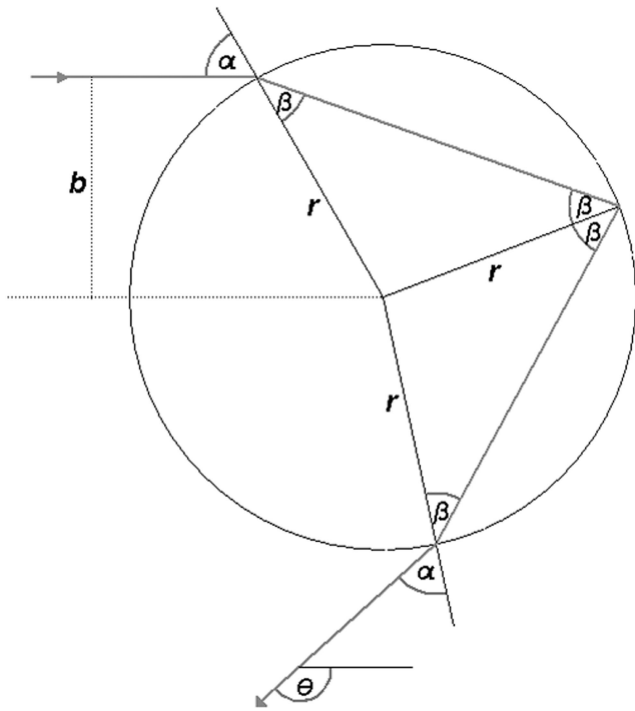


Fig. 3. Path geometry for  $p = 2$  scattering by a sphere: the incident ray is deflected by angle  $\theta = 180^\circ + 2\alpha - 4\beta$ . Note that  $b$  is the dimensionless impact parameter:  $b = 0$  corresponds to a central ray ( $\alpha = 0$ ), while  $b = 1$  and  $b = -1$  correspond to edge rays with grazing incidence ( $\alpha = 90^\circ$ ).

$r = 10 \mu\text{m}$  and  $n = 1.333$ . As the dimensionless impact parameter  $b = \sin \alpha$  increases from 0,  $\theta$  reduces from  $180^\circ$  until it reaches a minimum value at the primary rainbow angle of about  $137.9^\circ$  and then increases until  $\theta = 165.6^\circ$  when  $b = 1$  (grazing incidence). Such calculations show that light scattering in the directions of interest for glories, namely,  $\theta > 170^\circ$ , can be produced by near-central  $p = 2$  rays (i.e.,  $|b| < 0.2$ ). However, the scattered light generated by such rays is not responsible for the glory: calculations for the scattering of sunlight by these

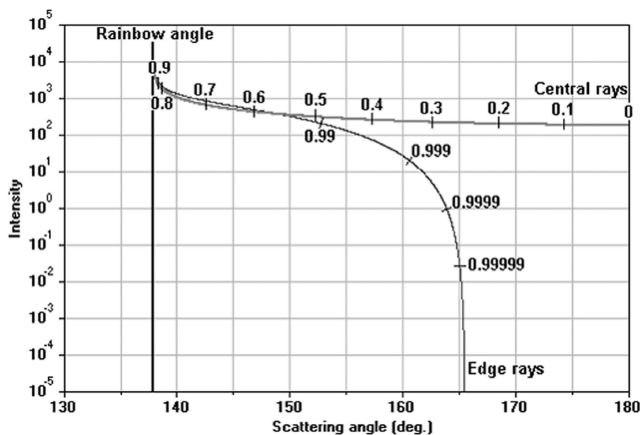


Fig. 4. Graph of intensity versus scattering angle  $\theta$  for geometric  $p = 2$  rays with refractive index  $n = 1.333$  (the values marked along the curves correspond to the impact parameter  $b$ ).

rays results in relatively uniform white light for  $\theta > 170^\circ$ . Note that, according to geometric optics, the intensity of the scattered light falls dramatically when  $b > 0.999$  (corresponding to rays incident near the edge of the sphere).

It is important to differentiate between backscattering and the glory. Numerous geometric ray paths result in precise backscattering (i.e.,  $\theta = 180^\circ$ ). Geometric considerations predict a significant enhancement in the intensity of the scattered light as  $\theta \rightarrow 180^\circ$ , but geometric optics does not offer any explanation for the formation of the rings of the glory, which centers on the antisolar point  $\theta = 180^\circ$ .

As illustrated in Fig. 5, van de Hulst<sup>9,10</sup> suggested that the glory is caused by  $p = 2$  rays with  $b = 1$  and postulated that the  $14.4^\circ$  gap between  $\theta = 165.6^\circ$  and  $\theta = 180^\circ$  could be bridged by surface waves. For simplicity, Fig. 5 shows the gap of  $14.4^\circ$  as the last part of the ray path before it emerges from the sphere, but this gap could be covered by three separate segments of surface waves covering a total of  $14.4^\circ$ . Because of symmetry of the sphere, ray paths of the form shown in Fig. 5 generate a toroidal wavefront, with a diameter nearly equal to that of the sphere, propagating in the direction  $\theta = 180^\circ$ . van de Hulst explained the glory as the interference pattern corresponding to this toroidal wavefront.

Fahlen and Bryant<sup>11</sup> conducted an experiment in which a water drop of 1.23 mm diameter was suspended in a sound field and illuminated by a He-Ne laser. They reported that, when the drop was viewed from  $\theta \approx 180^\circ$ , bright patches of light appeared near the center of the droplet (probably corresponding to light reflected from the front and back of the water drop for  $b \rightarrow 0$ ) and around the circumference of the sphere. Although the latter observation is consistent with van de Hulst's explanation, it does not provide

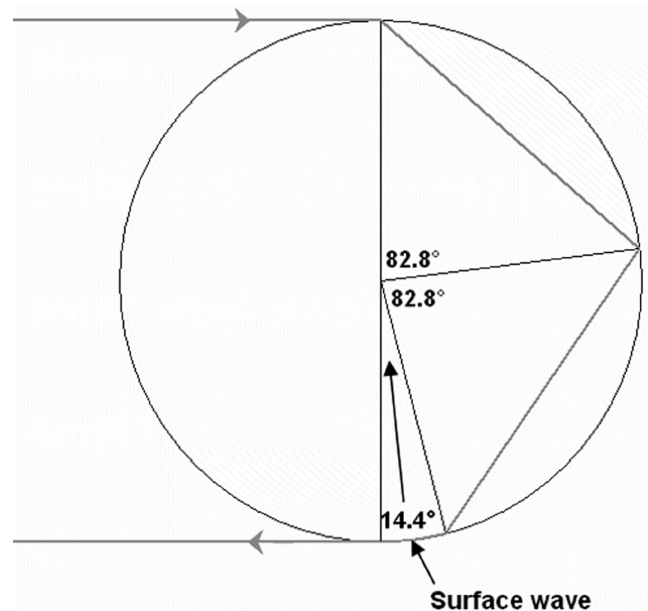


Fig. 5. van de Hulst's surface wave associated with a  $p = 2$  ray for a sphere with refractive index  $n = 1.333$ .

any information about the physical nature of the glory-making rays or about the actual role of surface waves.

The pioneering work of van de Hulst has been endorsed and extended by many other authors.<sup>12–15</sup> A major contribution to our understanding of glories has been made by Nussenzweig<sup>16–20</sup> who, for example, used the Debye series to analyze the relative importance of various scattering processes in the formation of the glory. In particular, Nussenzweig's results<sup>18,20</sup> indicate that  $p = 2$  contributions are dominant for droplets with effective radius  $x = 2\pi r/\lambda = 150$ , where  $\lambda$  is the wavelength of the light (corresponding to  $r \approx 13 \mu\text{m}$  for scattering of white light). This finding is consistent with the results shown in Fig. 1 of this paper, but Nussenzweig also reported that  $p = 11$  contributions become dominant for droplets with  $x = 500$  and  $x = 1500$  (corresponding to  $r \approx 44 \mu\text{m}$  and  $r \approx 130 \mu\text{m}$ , respectively). As discussed in the companion paper<sup>4</sup> in this issue, atmospheric glories are hardly visible when  $r > 30 \mu\text{m}$ . In any event, MiePlot calculations for  $r = 44 \mu\text{m}$  indicate that  $p = 2$  contributions are dominant in causing the rings of glory. The reason for this discrepancy is probably that Nussenzweig's calculations were made for  $\theta = 180^\circ$  (i.e., only for the central feature of the glory), while the MiePlot calculations cover the glory's rings as well as the central feature.

Nussenzweig has also drawn attention to the importance of resonances in forming the glory; for example, in 2003, he wrote<sup>21</sup> *"Tunneling is the dominant effect in backscattering. It produces the meteorological glory . . . The glory provides direct and visually stunning experimental evidence of the importance of resonances and light tunneling in clouds."*

Despite the success of van de Hulst's theory for the formation of the glory, various authors have bemoaned the lack of a simple physical model for the mechanisms causing the glory. For example, Lynch and Livingston<sup>22</sup> remark: *"Although the glory pattern is correctly predicted by Mie theory, a good physical explanation is, in our opinion, lacking. In some way light is backscattered after traversing the periphery of the droplet. Examined in detail, each drop is found to shine uniformly around its edge with an annulus of light that is coherent (the waves are in phase)." Greenler<sup>23</sup> notes: "In one sense, the glory is now well understood. A mathematical theory (Mie scattering theory) enables us to calculate the intensity variation in the glory pattern. Unfortunately, it gives us little physical insight into the process that produces the rings. . . . I wonder if there is no simple model containing the physical essence of the glory."* Similarly, Bohren and Huffman<sup>24</sup> state: *"Unlike the rainbow, the glory is not easy to explain, other than to say that it is a consequence of all of the thousands of terms in the scattering series, a correct but unsatisfying statement."*

In a paper<sup>25</sup> entitled "Does the glory have a simple explanation?" Nussenzweig tried to respond to the above requests, but he concluded that *"Mie theory describes the glory by the sum of a large number of*

*complicated terms within which the physical mechanisms cannot be discerned. CAM [Complex Angular Momentum] theory brings out the dominant physical effects and provides an accurate representation for each of them. That it does so by analytic continuation seems inevitable. I know of no other way of quantitatively representing tunneling."*

Despite this discouraging conclusion, it is perplexing that the glory cannot be explained (even by eminent scientists) except by "scientific arm waving." The rest of this paper reports attempts to quantify the scattering contributions made by surface waves, which seem to be the key process in the formation of glories.

#### 4. Surface Waves

Surface waves in optics may seem slightly mysterious. Nevertheless, the existence of surface waves in other areas of electromagnetic wave propagation is not in doubt: for example, vertically polarized radio transmissions at frequencies around 1 MHz propagate via "ground waves."

Unfortunately, no rigorous method seems to be available for calculating the intensity of scattering caused by surface waves. Various authors<sup>7,20,26,27</sup> have proposed approximate methods applicable to scattering of light by small spheres, but all warn that their approximations are not valid at  $\theta = 180^\circ$  or, indeed, near  $180^\circ$ . These limitations are especially problematic for investigations of the glory! Although these approximate methods are fairly similar, the surface-wave calculations reported in this paper are based on Khare's method.<sup>26</sup>

As the scattering contributions made by surface waves are generally much weaker than those due to other scattering mechanisms, experimental verification is obviously difficult. On the other hand, Debye-series calculations can be used to isolate specific scattering processes even if they do generate only very weak scattering. For example, the Debye series  $p = 1$  term accurately defines transmission through a sphere: for small values of  $\theta$ , the Debye-series term closely matches calculations based on geometric optics, as shown in Figs. 6(a) and 6(b). As geometric optics cannot make any contribution to the  $p = 1$  scattered intensity when  $\theta > 180^\circ - 2 \sin^{-1}[1/n] \approx 82.8^\circ$  for  $n = 1.333$ , another mechanism must be responsible for scattering in these directions.

Figure 6(a) and 6(b) show that Khare's calculation method for surface waves gives a good approximation to the Debye-series term for  $p = 1$  when  $\theta > 82.8^\circ$ . There is a slight error in intensity in Fig. 6(b) (for  $r = 10 \mu\text{m}$ ), but the differing slopes of the curves for the two polarizations are correctly reproduced. This close agreement confirms that surface waves are the dominant  $p = 1$  scattering mechanism for  $\theta > 82.8^\circ$ . As surface waves shed light continuously as they travel along the surface of the droplet, the intensity of the scattering due to surface waves reduces exponentially with the length of the path taken by the surface wave, thus explaining why the intensity of

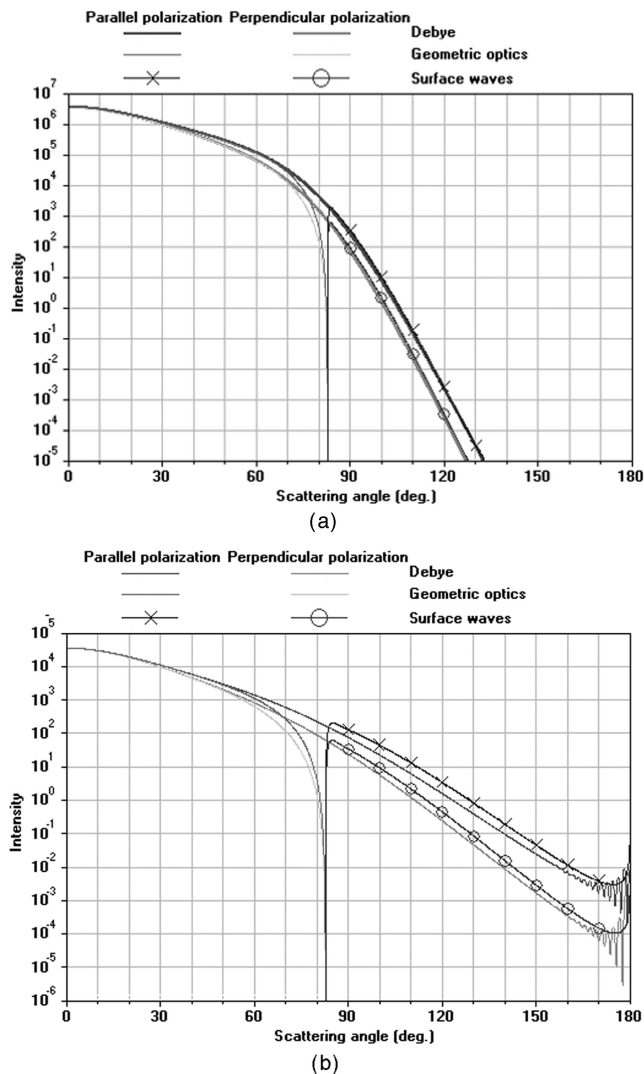


Fig. 6. (a) Comparison of calculation methods for  $p = 1$  scattering of light of wavelength  $\lambda = 650$  nm from a sphere of radius  $r = 100 \mu\text{m}$  and refractive index  $n = 1.333$ . (b) As in (a) but with  $r = 10 \mu\text{m}$ .

the surface waves reduces much more rapidly for  $r = 100 \mu\text{m}$  [Fig. 6(a)] than for  $r = 10 \mu\text{m}$  [Fig. 6(b)]. This exponential decay for surface waves can also be recognized by the fact that the “curves” of intensity versus  $\theta$  in Figs. 6(a) and 6(b) are almost straight lines (because the intensity scale is logarithmic while the angular scale is linear). In general, the surface-wave intensity due to parallel polarization is significantly higher than that due to perpendicular polarization. As most natural glories seem to be caused by scattering from water droplets with  $r$  between  $4 \mu\text{m}$  and  $25 \mu\text{m}$ ,<sup>4</sup> the rest of this paper will use examples based on the scattering of red light ( $\lambda = 650$  nm) from  $r = 10 \mu\text{m}$  water droplets.

Note that the Debye-series term in Fig. 6(b) shows a series of maxima and minima as  $\theta \rightarrow 180^\circ$ . What causes these ripples? The top part of Fig. 7 shows a ray with an impact parameter  $b = 1$  taking a shortcut through  $82.8^\circ$  and then propagating  $92.2^\circ$  clockwise

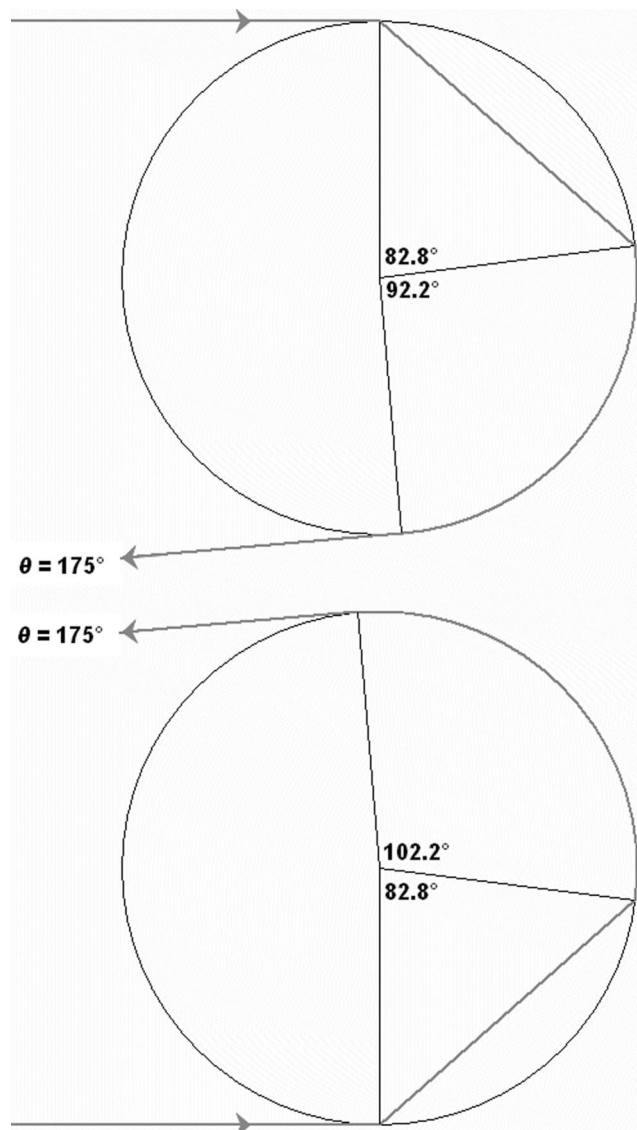


Fig. 7. Surface-wave paths ( $p = 1$ ) resulting in scattering angle  $\theta = 175^\circ$ : the upper part of this diagram shows the “short” path, while the lower part shows the “long” path.

along the surface of the sphere, resulting in a scattering angle  $\theta = 82.8^\circ + 92.2^\circ = 175^\circ$ . The lower part of Fig. 7 shows a ray with  $b = -1$  taking a shortcut through  $82.8^\circ$  and then propagating anticlockwise  $102.2^\circ$  along the surface of the sphere, resulting in deflection of  $82.8^\circ + 102.2^\circ = 185^\circ$ , which is equivalent to scattering angle  $\theta = 360^\circ - 185^\circ = 175^\circ$ . Of course, the value of  $\theta = 175^\circ$  in Fig. 7 has been chosen solely as an illustrative example. More generally, scattering in a specific direction  $\theta = 180^\circ - \delta$  can be caused by two surface-wave components: the “short” path generated by incident rays with impact parameter  $b = 1$  involving a deflection of  $180^\circ - \delta$  and the “long” path generated by incident rays with impact parameter  $b = -1$  involving a deflection of  $180^\circ + \delta$ .

Figure 8(a) shows in greater detail the ripples on the Debye  $p = 1$  term, together with calculations of

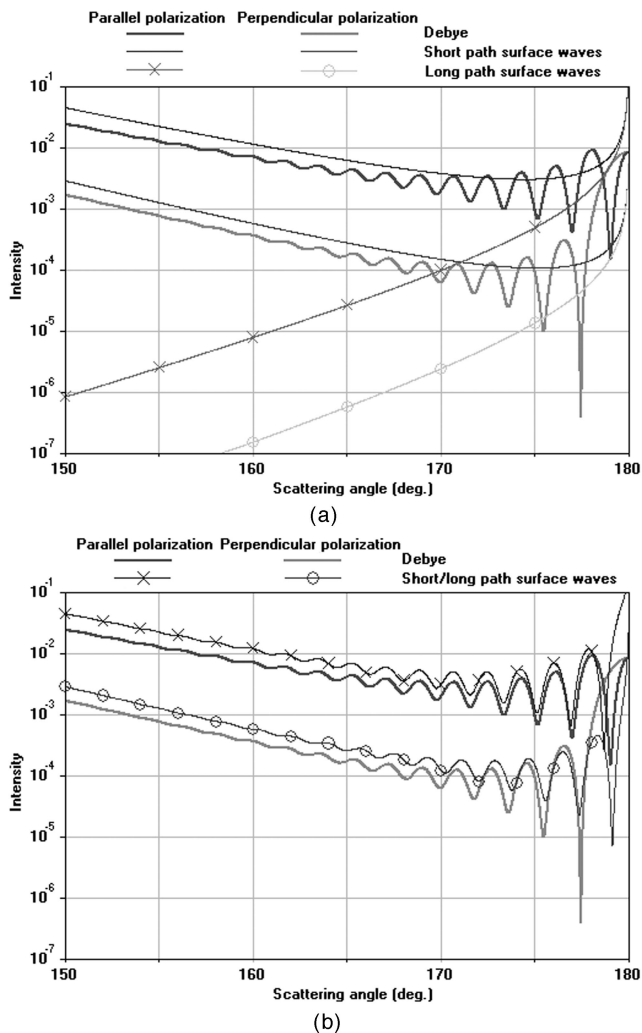


Fig. 8. (a) As in Fig. 6(b), but showing the separate contributions from short-path and long-path surface waves (see Fig. 7). (b) As in (a), but showing the vector sum of the short-path and long-path surface wave contributions.

the intensity due to the short- and long-path surface waves. The long-path contributions are weaker than the short-path contributions simply because the longer path gives greater attenuation. The difference in path length between the long path and the short path also causes a phase difference between the two contributions. Constructive interference will occur when this phase difference is a multiple of  $360^\circ$ , whereas destructive interference will occur when it is an odd multiple of  $180^\circ$ . Consequently, a series of maxima and minima will occur in the scattering pattern as a function of  $\theta$ . Note that there is an additional phase shift of  $90^\circ$  between the short and long paths because the long path crosses one focal line more than the short path.<sup>28</sup>

Figure 8(b) compares the Debye  $p = 1$  term with the vector sum of the contributions from the short- and long-path surface waves. The general shape of the ripples in Fig. 8(b) is reassuringly similar to the Debye-series calculation. Note that Fig. 8(a) provides an explanation for the increasing amplitude of the

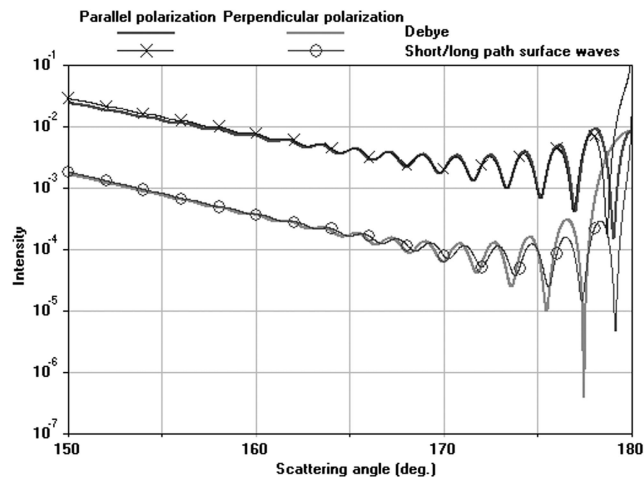


Fig. 9. Comparison of Debye-series and surface-wave calculations for parallel polarization for  $p = 1$  scattering (N.B., Amplitude from Khare's formula multiplied by factor of 0.8).

ripples as  $\theta \rightarrow 180^\circ$  in Fig. 8(b). When  $\theta \approx 150^\circ$ , Fig. 8(a) indicates that the short-path surface wave is dominant, and hence there are no ripples in this part of Fig. 8(b). However, as  $\theta$  is increased toward  $180^\circ$ , Fig. 8(a) shows that the intensity of the long-path surface wave increases relative to the short-path surface wave, and consequently the ripples in Fig. 8(b) become larger. Note that these ripples correspond to the circular rings of a glory caused by  $p = 1$  scattering. This “mathematical” result may be surprising, but  $p = 1$  glories cannot be observed in practice because the intensity of the  $p = 2$  glory is more than 5 orders of magnitude greater. Nevertheless, as surface waves are the only  $p = 1$  scattering mechanism applicable to  $\theta > 82.8^\circ$ , the Debye  $p = 1$  term provides a crucial test of the accuracy of surface-wave calculations.

Khare's approximation overestimates the intensity of surface waves and actually gives infinite intensity for  $\theta = 180^\circ$ . Figure 9 shows that excellent agreement with the Debye-series calculations for  $p = 1$  scattering (at least for  $\lambda = 650$  nm and  $r = 10$   $\mu\text{m}$ ) can be achieved if the amplitude calculated by Khare's for-

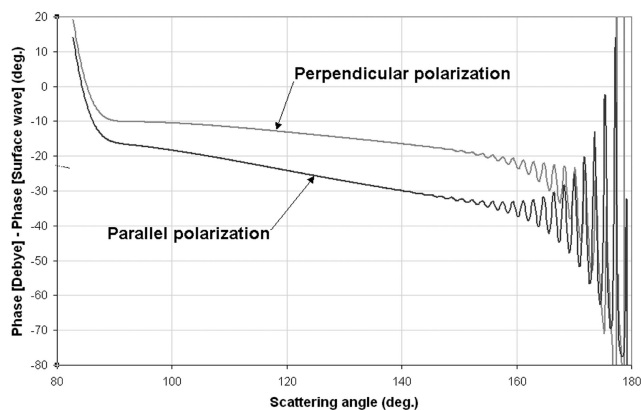


Fig. 10. Difference in phase between the Debye-series and the short-path surface-wave calculations for  $p = 1$  scattering.

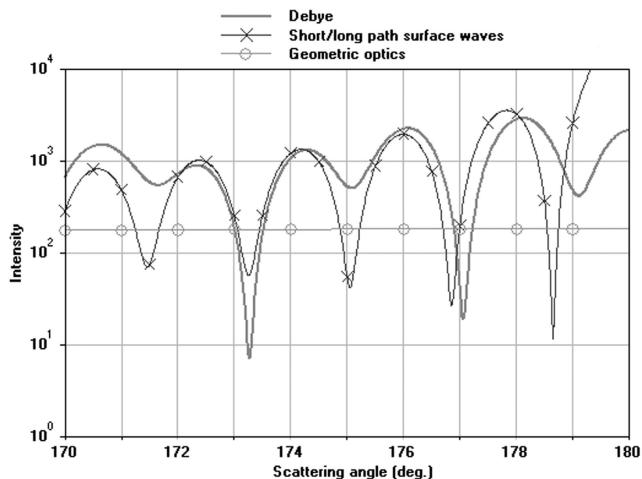


Fig. 11. Comparison of Debye-series, surface-wave, and geometric-optics calculations for parallel polarization for  $p = 2$  scattering (N.B., Amplitude from Khare's formula multiplied by factor of 0.5).

mula is multiplied by an arbitrary value of 0.8. As the precise locations of the maxima and minima are slightly different, especially for perpendicular polarization, this suggests that there are some phase differences inherent in the approximation. Figure 10 compares the absolute phase of the Debye  $p = 1$  term with that of Khare's approximation for the short-path surface wave. The extreme fluctuations in Fig. 10 as  $\theta \rightarrow 180^\circ$  are because the Debye term is the vector sum of the short- and long-path surface waves, whereas the surface-wave term is restricted to the short path.

The discussion about surface waves has so far focused on  $p = 1$  scattering because this facilitates calibration of the approximate calculation methods for surface waves against the rigorous Debye-series calculations. As indicated in Section 2,  $p = 2$  scattering is responsible for the colored rings of the natural glory. It is thus necessary to extend the analysis of surface waves to cover the  $p = 2$  case, as in Fig. 11, which compares the Debye-series calculations with the surface-wave calculations. To take account of the fact that Khare's method overestimates the intensity of the surface waves, the amplitude predicted by Khare's formula has been multiplied by an arbitrary factor of 0.5. Although the curves for the Debye series and for surface waves in Fig. 11 are broadly similar, there are significant differences in detail. The principal reason is the existence of a further  $p = 2$  scattering mechanism that is due to near-central rays (i.e., for  $|b| < 0.2$ ). As illustrated in Fig. 12, there are three separate ray paths that, for refractive index  $n = 1.333$ , result in  $\theta = 175^\circ$ . Figure 11 includes a curve showing the intensity of this geometric optics  $p = 2$  term. The scattered intensity can be calculated by taking the vector sum of all three contributions, but, as mentioned above, there are errors in the absolute phase given by the approximate calculations for surface waves. In the particular case of  $\lambda =$

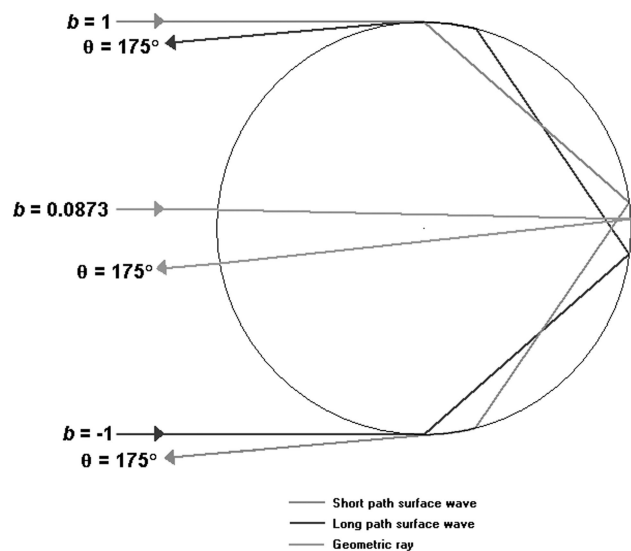


Fig. 12. Diagram showing  $p = 2$  rays for that result in a scattering angle  $\theta = 175^\circ$  for a sphere with refractive index  $n = 1.333$ .

650 nm and  $r = 10 \mu\text{m}$ , the phase error for  $p = 2$  scattering when  $\theta \approx 180^\circ$  is estimated to be about  $40^\circ$ . Figure 13 shows the vector sum of the geometric-optics contribution and of the short- and long-path surface-wave contributions (adjusted by a factor of 0.5 in amplitude and by  $+40^\circ$  in phase). Although Fig. 13 indicates that inclusion of the geometric-optics contributions improves the agreement with the Debye  $p = 2$  results, the approximate nature of Khare's formula remains a substantial limitation to more detailed analysis.

## 5. Discussion

All of the results in Section 4 are based on approximate calculation methods for intensity of surface waves. As such methods produce errors in terms of

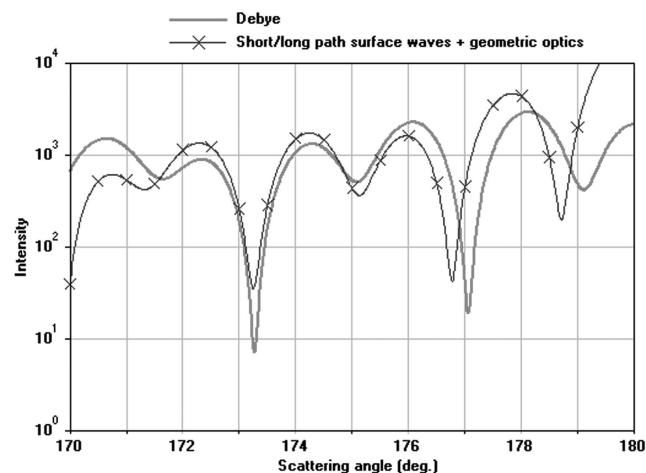


Fig. 13. Comparison of Debye-series calculations with the vector sum of surface-wave and geometric-optics calculations for parallel polarization for  $p = 2$  scattering (N.B., Amplitude from Khare's formula multiplied by factor of 0.5 with a phase correction of  $+40^\circ$ ).

intensity and phase, a more accurate calculation method is needed. Despite such errors, the results suggest that the colored rings of the glory are due to interference between short- and long-path surface waves associated with  $p = 2$  contributions.

At first sight, this explanation may seem similar to that of van de Hulst, but it is instructive to examine the differences between the two explanations. Using Huygens' principle, van de Hulst showed that the interference pattern corresponding to the toroidal wavefront propagating in the direction  $\theta = 180^\circ$  is defined by

$$I_1 = [C_1 \{J_1(u) - J_2(u)\} + C_2 \{J_1(u) + J_2(u)\}]^2,$$

$$I_2 = [C_2 \{J_1(u) - J_2(u)\} + C_1 \{J_1(u) + J_2(u)\}]^2,$$

where  $I_1$  and  $I_2$  are the intensities in the direction  $\theta = 180^\circ - \delta$  for perpendicular and parallel polarization respectively,  $C_1$  and  $C_2$  are proportional to the amplitudes of the components with perpendicular and parallel polarization, and  $u = 2\pi r \sin(\delta)/\lambda$ . In assessing the accuracy of van de Hulst's method, an immediate problem is that we do not know the values of  $C_1$  and  $C_2$ . Although van de Hulst suggested some notional values, such as  $0 > C_1/C_2 > -0.25$ , the above equations do not yield any quantitative predictions of the intensity of the glory.

Figure 14(a) shows a comparison of the Debye series for the  $p = 1$  glory with van de Hulst's method: in this case, the value of  $C_1/C_2 = -0.12$  has been chosen because it reproduces the minimum for perpendicular polarization at  $\theta \approx 177.4^\circ$ , while the value of  $C_1$  has been set to give the correct intensity at  $\theta = 180^\circ$ . Figure 14(a) shows that van de Hulst's diffraction pattern agrees closely with the Debye-series calculations when  $\theta$  is near  $180^\circ$ , but becomes increasingly inaccurate as  $\theta$  is reduced.

Figure 14(b) compares the Debye series for the  $p = 1$  glory with the calculation method based on simple interference between short- and long-path surface waves. The latter method gives inaccurate results when  $\theta \rightarrow 180^\circ$ , but it correctly reproduces the maxima and minima for other values of  $\theta$ , especially for the dominant parallel polarization.

In essence, Fig. 14(a) shows that van de Hulst's diffraction pattern is not a satisfactory model for the rings of the glory, whereas Fig. 14(b) and Fig. 8(a) act as reminders that surface waves shed light continuously as a function of  $\theta$ , unlike van de Hulst's diffraction pattern that is based on  $\theta = 180^\circ$  being a preferential direction.

The fact that the Debye-series calculations predict a glory for the  $p = 1$  term, as well as for the  $p = 2$  term, may be unanticipated. Figure 1 shows that many other values of  $p$  produce almost identical color sequences for the scattering of sunlight, although they are much weaker than the  $p = 2$  term. Identification of the scattering mechanisms responsible for these "invisible" glories would be aided by a rigorous calculation method for the intensity of surface waves.

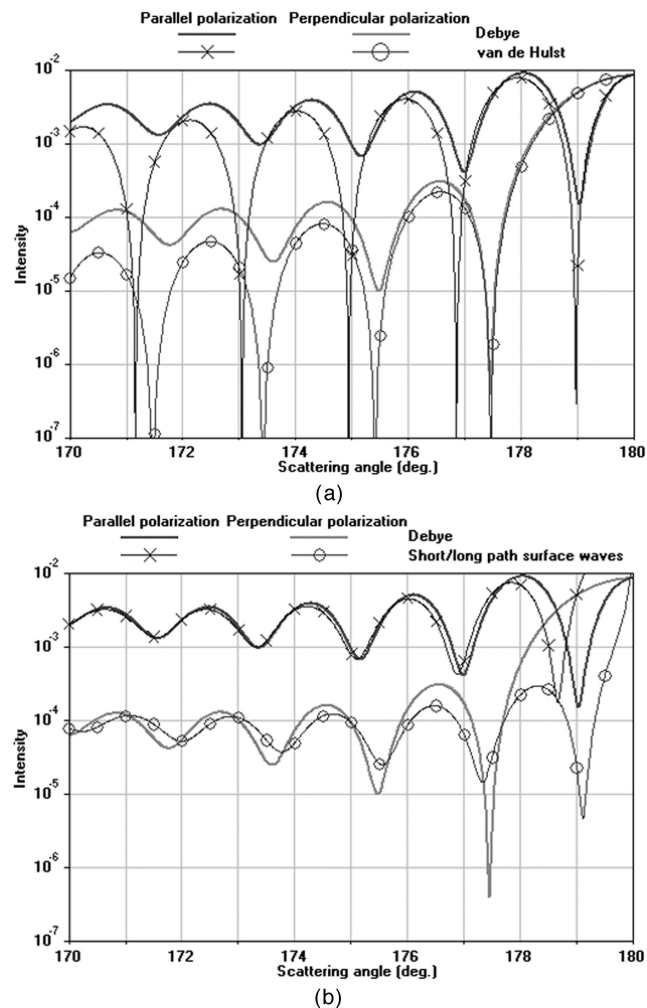


Fig. 14. (a) Comparison of Debye-series calculations for  $p = 1$  scattering with calculations based on van de Hulst's diffraction pattern. (b) Comparison of Debye-series calculations for  $p = 1$  scattering with calculations based on two-ray interference between short- and long-path surface waves.

## 6. Conclusions

Mie theory can be used to generate full-color simulations of the atmospheric glory. However, as Mie theory involves the summation of a large number of numerical terms, it does not provide any explanation of the scattering mechanisms causing the glory. Calculations using the Debye series provide a partial explanation by showing that  $p = 2$  scattering (i.e., rays suffering one internal reflection within spherical water droplets) are dominant in the formation of the colored rings of the glory. Higher-order rays ( $p > 2$ ) contribute little to the colored rings but make important contributions to the white central feature of the glory.

This paper suggests that the colored rings of the glory are caused by two-ray interference between "short" and "long" path surface waves, which are generated by rays entering the droplets at diametrically opposite points, as illustrated in Fig. 12. It is also

necessary to take account of geometric “near-central”  $p = 2$  rays.

van de Hulst postulated that the glory is caused by the interference pattern corresponding to the toroidal wavefront propagating in the direction  $\theta = 180^\circ$ . This paper emphasizes that it is not necessary to invoke  $\theta = 180^\circ$  as a preferential direction because surface waves radiate in all directions. Furthermore, van de Hulst’s method does not accurately predict the maxima and minima corresponding to the rings of the glory, whereas calculations based on interference between short- and long-path surface waves seem much more accurate.

As this paper’s explanation of the glory does not rely on summation of numerical terms (such as in Mie theory or the CAM approximation), it is accessible to nonmathematicians and, consequently, it may satisfy the many requests for a simple physical model of the glory.

The author would like to thank G. P. Können for his generous advice and encouragement.

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